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Case Study of an Online Course on *Introductory Mathematics for Artificial Intelligence*[*]

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Abstract

Artificial Intelligence (AI) is one of the most crucial factor affecting future career choice of students. Accordingly, we offered students a semester-long Introductory Mathematics for Artificial Intelligence course. The course was developed to provide undergraduate students with various majors to navigate their career opportunities in AI-related jobs. This paper presents the newly published textbook and an online mathematics laboratory that integrated effective technology for this introductory course. In addition, we introduce a process-driven evaluation method based on the Problem/Project-Based Learning (PBL) report that worked well for this course. We also share trails of communications with our students and added the results of our students' performance analysis using principal component analyses (PCA) with the data generated from the class.

1. Introduction

Artificial intelligence (AI) is being actively used almost everywhere. For example, military, finance, medical, legal, and industrial sectors use AI, especially deep-learning and machine learning algorithms. Demand for AI experts has drastically increased. Many universities endeavor to educate students of diverse backgrounds (e.g., science, engineering, arts, social science, sports science, law, etc.) to understand and utilize AI algorithms to meet their fields' needs.

While it is difficult and meaningless to define using AI for any task in one word [10], it often requires an ability to process and predict using given data. In this process, various mathematical theories and

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techniques are used. It is not trivial to apply AI appropriately to solve problems without studying mathematics and understanding the principles of algorithms.

This study first developed a curriculum covering essential mathematics concepts needed for AI for university students of various majors. Then we offered a three credits course in the spring semester of 2020. Finally, we published a textbook *Introductory Mathematics for AI*, an e-book (https://buk.io/@kb5299). We also created and provided an online mathematics laboratory (http://matrix.skku.ac.kr/intro-math4ai/). The textbook and the laboratory are the novelty of our work.

Of course, there are online courses[‡] and books [5,26] on mathematics for artificial intelligence, machine learning, and data science. However, unlike these existing online courses and books, we needed a new curriculum that meets the following four conditions; First, it should be easy enough for all college freshmen to follow. In other words, it was necessary to carefully select the most of core contents so that students in any major could have a big pictue of mathematics used in artificial intelligence. This is because this course aims for equity in AI literacy[§]. Second, we hoped that the learning of mathematics should be core part of this curriculum over artificial intelligence in a convergence approach [11,31]. Third, the emphasis was placed on applying mathematical concepts through hands-on practice, rather than simply learning and solving problems. The paradigm of learning has shifted to one that can be applied rather than just knowing [18,27], and textbooks and materials should be able to support these things [24,30]. Therefore, it was hoped that the students would directly utilize the convergence of mathematics and artificial intelligence and contribute to making them into their own knowledge. Finally, it was our aim to provide an environment in which active interactions can occur in all these series of learning processes as is suggested in [3]. These are differences of our efforts from the existing online courses and books.

Due to the COVID-19 pandemic, we had to offer this course online. However, in general, online classes have several limitations. In particular, it is challenging to grasp students' learning process, induce rigorous thinking and learning through discussions, give fair evaluations, and design out-of-class activities [2,6,20]. In addition, we cannot emphasize enough that students need to adapt themselves to the educational paradigm shift, and we need to lead educational innovation in such a rapidly changing social environment [23].

To overcome these difficulties of online classes, the researchers made the following instructional design and conducted the classes accordingly. First, an environment was established to promote students' self-activities within and outside the class. To do this, we provided a new online mathematics laboratory, that was built with pre-coded Python-based SageMath cells. Students could practice what they learned by modifying the code quickly and trying various data analysis tasks in this online laboratory. Students had access to pre-recorded lectures in the university's Learning Management System (LMS) at least a week in advance of real-time streaming lectures to study the teaching material beforehand and ask any questions that arise during their studies. Additional live video streaming Office Hours (OH) were offered through Zoom or WebEx once or twice a week. The

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‡ AI Mathematics - W3Schools. https://www.w3schools.com/ai/ai_mathematics.asp

Mathematics for Machine Learning. https://www.coursera.org/specializations/mathematics-machine-learning

§ Artificial Intelligence for All: A Call for Equity in the Fourth Industrial Revolution.

https://ourworld.unu.edu/en/artificial-intelligence-for-all-a-call-for-equity-in-the-fourth-industrial-revolution
The Artificial Intelligence (AI) for K-12 initiative (AI4K12) https://ai4k12.org/

students had opportunities to ask questions, get answers from the instructor, and discuss with their classmates during live video streaming lectures.

Second, we developed new and straightforward True/False-type quiz questions. These were automatically evaluated so the instructors could know whether the student faithfully participated in the learning process. Finally, students organized what they had learned, discussed it in the learning process, and included these outputs in their portfolios (Problem/Project-Based Learning (PBL) report). PBL report links to http://matrix.skku.ac.kr/2021-Math4AI-Fall-PBL/.

Third, we diligently provided answers to students' questions for vital communications, mainly through the Q&A board on our LMS. In addition, we used the last half of each online OH to allow students to explain the open problems that arose so that the instructor could add the final comments on them.

Fourth, we incorporated a flipped learning method [12] to influence students' deep thinking and learning. Students viewed pre-recorded lecture videos in advance. Those were very helpful since students could repeatedly study at their convenience. In addition, students could ask questions on the Q&A board and get answers from the instructor and classmates.

We notified students in advance of a process-based evaluation that includes a report presentation, a Q&A session, and a video presentation. Finally, we also conducted peer evaluation to ensure the evaluation process was fair in many aspects. Students' interim scores were posted once every two weeks.

In this study, various efforts were made to overcome the limitations of online classes. In addition, we aimed to explore how students perceived their growth of skills and knowledge of AI. We were able to have very positive outputs from this newly developed course through these experiments and efforts.

Our contribution to mathematics education for AI is that we developed a new process-driven teaching/learning method using a textbook, a lab, and an evaluation method for our students. In particular, our study combined various teaching and learning methods to help students acquire the most vital knowledge in understanding AI algorithms. We like to share the results and principal component analysis (PCA) of our experiment.

2. Mathematics for artificial intelligence

This course aimed to let learners understand how basic mathematical knowledge is used when developing and using AI. Students learn the basic mathematics content necessary to understand AI and apply it to problems each student faces. To achieve these goals, we divided the textbook's contents into four chapters. Chapters 1 to 3 dealt with linear algebra, multivariate calculus, and statistics. In Chapter 4, PCA and backpropagation, which are essential in machine learning and data analysis, is explained using the concept from Chapter 1 to 3.

We selected mathematics topics in Chapters 1 to 3 after discussions with three instructors from the School of Artificial Intelligence at Sungkyunkwan University. We included only the essential concepts. These topics included singular value decomposition (SVD) in Chapter 1, gradient descent method in Chapter 2, and methods to obtain the covariance matrix from given data in Chapter 3. Chapter 4 included PCA and artificial neural networks (ANN) along with real-life examples so that students could understand why and how to use AI in real-life. Table 1 shows the selected essential mathematics topics we included in the textbook.

Table 1: *Essential mathematics topics for AI*

Mathematics Subjects	**Essential Mathematics Topics**
Linear Algebra	Orthogonal Projection, Shortest Distance, Linear System of Equations, Least Squares Solution, Eigenvalues, Matrix Diagonalization, Singular Value Decomposition
Multivariate Calculus	Limits, Derivatives, Multivariate Functions, Partial Derivatives, Gradients, Extreme Values of Functions, Gradient Descent Method
Statistics	Counting Methods, Probability, Random Variables, Probability Distributions, Covariance, Correlation, Covariance Matrices
PCA & ANN	Principal Component Analysis, Artificial Neural Network, Backpropagation, Machine Learning (ML), Deep Learning

The above mathematics topics and Python/R codes are covered in a 15-week semester course, including mid-term and final exams. In particular, we developed and provided an online mathematics laboratory in Python (Python/SageMath) and R so that students could not only understand mathematical principles but also compute with provided codes. Moreover, it gave a specific and convenient way to understand real-life data analysis (Figure 1).

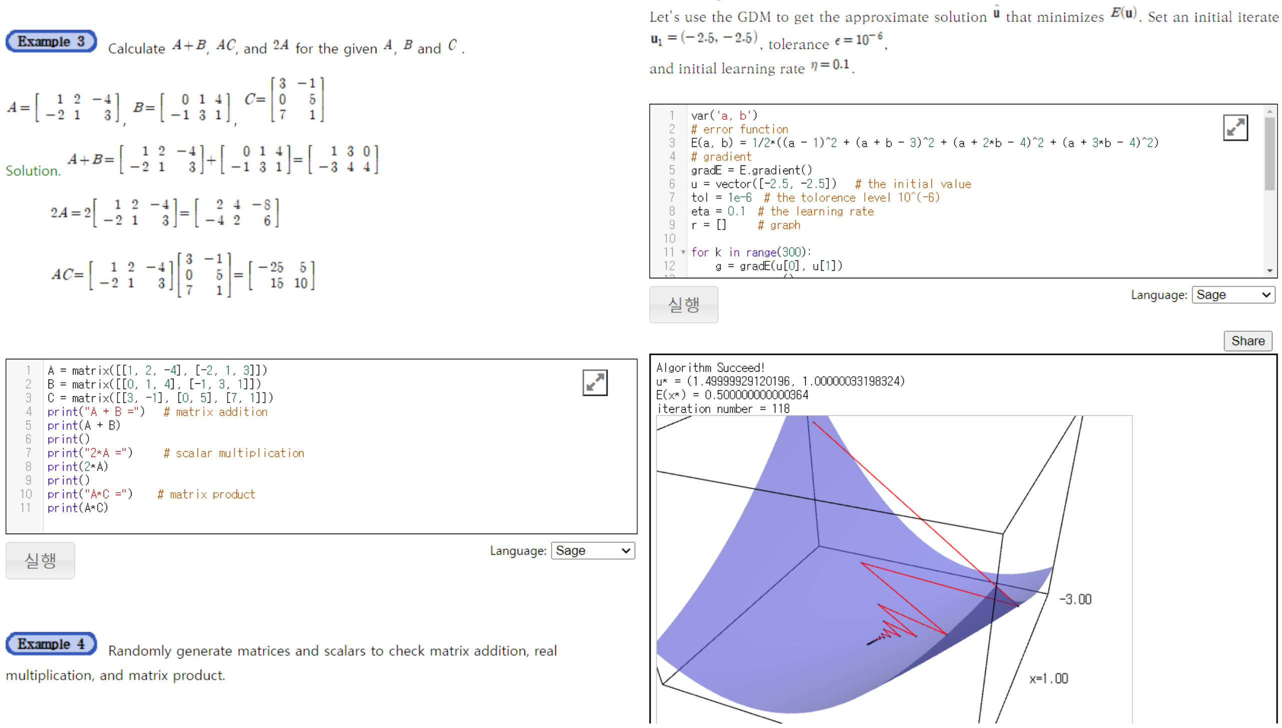

Figure 1: *Screenshots from the online laboratory of our course*

This course is designed for first-year students [20]. The final version of our teaching material can be found at http://matrix.skku.ac.kr/intro-math4AI/. The link leads to sample lecture videos, lecture notes, and online lab. Some can be found in the Appendix.

3. Teaching-learning methods for online classes

Online learning has accelerated in Korea, especially during the COVID-19 pandemic. It has the advantage of no limit in time and places. However, it has shown clear disadvantages in monitoring students' academic activities, communicating with students, and understanding students' achievement from the instructors' perspectives. More importantly, students without self-learning experiences suffered from learning efficiency [2].

We designed the course differently to overcome such obstacles using online content, video lectures, online meetings using Zoom and/or WebEx, and frequent communication using a Q&A board. In addition, students often used their mobile devices. Moreover, various teaching and learning methods were introduced to support the interactive learning process to overcome the limitations of the online learning environment.

3.1 Flipped learning to support level-specific learning

Flipped learning can motivate students to learn through pre-class resources and acquire knowledge according to their level through self-learning [12]. For example, in our flipped learning format course, we added an e-book and lab to help students not write/type code to practice (Appendix).

3.2 Portfolio to promote self-learning management

Previous research has shown that making a course portfolio by a student in mathematics learning is a powerful tool. A course portfolio can be used to present, create new content, publicize own ideas, and learn mathematics in different ways [19].

We asked students to include all learning activities in their portfolio (PBL report) in our course. The required forms and samples for students' portfolios were provided on our class website http://matrix.skku.ac.kr/PBL-Form/PBL-Report-Form-English.docx. Students downloaded those to create their portfolios and presented them twice, during the midterm and final exam periods. Their class portfolios were then peer-reviewed.

We also asked students simple weekly questions about their progress and participation level, and students were instructed to include the answers to these questions in their portfolios, as Table 2 shows. Students could monitor their weekly class participation while answering True/False-type quiz questions. These diagnosed their learning process. Finally, students could reflect on their learning through these processes (e.g., Table 2).

Table 2: *Examples of students' reflection on the learning process*

1. I shared my introduction and motivation for taking this course in the Q&A.
2. I attended all classes in Weeks 1–2, and after completing classes in Weeks 1 and 2, I solved at least two problems each week. I shared them in the Q&A. I also commented, modified, or answered on more than two solutions from other students (or I prepared or reviewed class content more than four times and summarized them in a Q&A; or I added new information related to the content summarized by another student; or I added new information and comment to the content shared by the instructor more than four times).
3. I read Math4AI (http://matrix.skku.ac.kr/2020-AI-translation/) and commented on the Q&A before week 3 class.

3.3 Process-driven fair evaluation management

This study's process-driven evaluation focused on evaluating students' performance by monitoring their learning process. Such evaluation can contribute to process-driven learning, improve learning quality, and integrate instruction and evaluation [17,25]. Therefore, process-driven evaluation can solve the difficulty of fairness and secure the validity of the evaluation [20].

This course's process-driven evaluation included questions and answers (Q&A) in class, Q&A participation content, and discussions on the outcome of the project and presentations [20,22]. For example, students collected questions and answers shared on the Q&A board, discussions with instructors and classmates, and the discussion results in a portfolio. Then, the instructor conducted a qualitative/quantitative evaluation of all items in the portfolio as well as learning activities, such as discussions. In other words, the instructor evaluated qualitatively whether students had organized new key concepts well and evaluated their participation in Q&A and online quizzes quantitatively. The evaluation becomes fairer and more valid when a student's performance is evaluated from various angles.

60 points (%) were assigned to the mid-term and final online exams in this course. The remaining 40 points (%) were assigned to original ideas and well-organized content by evaluating students' portfolios.

3.4 LMS for reinforcing self-learning

The LMS provides the critical elements of self-learning competencies in online classes such as learning desire, goal setting, self-management, learning continuity, effort attribution evaluation, and self-reflection. The LMS is reported to play a vital role in the improvement of the learning process and achievement and has a positive effect on self-learning and class satisfaction [4,13,16].

This study integrated the LMS functions designed to improve students' self-learning capabilities. The LMS we used was provided by our institution's teaching and learning development center. The use of the LMS was intended to induce class participation, give self-reflection opportunities, and provide appropriate feedback from instructors. For example, we used the LMS's basic functions, such as syllabus, lecture materials, announcements, Q&A, quizzes, assignments, discussion, online exams, group activities, portfolio management, and submission. Moreover, all lecture content in this course was produced and uploaded in advance as text and video in the LMS to support students' learning and promote interaction. Finally, we added additional functionality to our LMS to efficiently copy and paste mathematical formulas from relevant teaching material and web content to answer the questions.

3.5 Q&A for revitalizing student-instructor interaction

The online Q&A is a crucial LMS function that assists the interaction between students and instructors [13]. Students can ask questions about content they do not understand, and instructors can answer them using the Q&A function. LMS's Q&A function is one of the tools used to present various learning content sources, change teaching methods, and provide the correct learning path [15,29]. Instructors can work out how much students have grasped the learning content by examining students' questions, providing an additional explanation, and adjusting instructional content through Q&A [32].

We utilized the Q&A function very well. Students posted questions about what they could not understand, and instructors then posted answers to those questions. Unfortunately, Korean students are often passive during classes. But they felt a lot more comfortable using the Q&A function. We asked students to post a summary of learning content and questions on top of their weekly oral presentations. Students were a lot more active on the Q&A board than in classes. Figure 2 is an example of students posting the content they had learned.

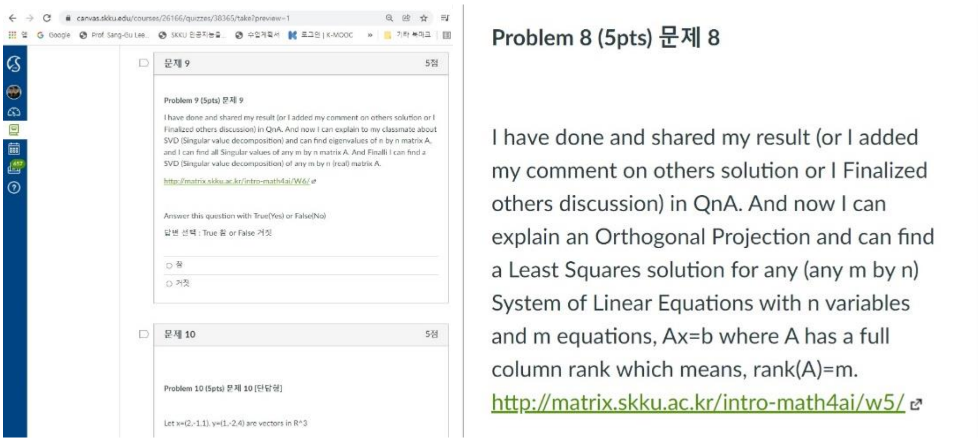

Figure 2: *Example of an online Q&A screen in which students participated*

The Q&A record helped the instructor identify concepts that students did not sufficiently understand and provided more explanations and instructions about those concepts by asking questions during online live classes.

3.6 Course management

This course reflected the various teaching and learning intentions discussed in Sections 3.1 to 3.5 to increase student participation and achievement. Therefore, we summarize our procedure as follows.

1. The instructor uploaded video lectures to the LMS.
2. Students undertook self-study using the uploaded contents before the class.
3. Students summarized the lecture and/or asked questions on the Q&A.
4. Students talked about content in Q&A during live face-to-face online meetings and the instructor gave more in-depth explanations.
5. Students were encouraged to finalize what they understood and add them to their portfolios.
6. The instructor offered quizzes on the LMS to check the activities of students.
7. Students prepared an individual portfolio (PBL report), including the outcomes from their learning activities, and presented it twice during the midterm and final exam periods. The presentation was subject to peer evaluation.
8. Students practiced computations and conducted data analysis projects using the online mathematics laboratory where simulation and calculation were possible.
9. We recommended that students finalize their portfolios and display a video clip of their final PBL presentation.
10. Students checked their interim scores individually and were notified periodically. It made students make more effort to reach their desired grades. By doing so, our course could connect evaluations and the learning process.

4. Analysis

4.1 Principal component analysis (PCA) of data from students' diverse activities
To analyze data generated throughout the course, we collected each student's scores with their final grade for the final PBL presentation/exam, portfolio, midterm exam, Q&A, attendance, and video presentation of the project. Figure 3 shows the part of our evaluation sheet.

Name	Final	PBL	Mid	Q&A	Atte	Project	Class	Grade
***	100	66	61	47	35	0	**2**	**B**
***	0	0	0	25	21	0	**1**	**F**
***	0	0	0	0	0	0	**1**	**F**
***	100	72	75	50	36	9	**3**	**B+**
***	100	65	80	50	36	0	**2**	**B+**
***	120	78	97	50	36	10	**3**	**A+**
***	0	0	0	3	3	0	**1**	**F**
***	0	0	0	24	21	0	**1**	**F**
***	120	77	98	50	35	7	**3**	**A+**
…	…	…	…	…	…	…	**…**	**…**

Figure 3: *Evaluation sheet*

This data consists of six features. First, we did PCA using the R program to reduce its dimensions. Our analysis results are shown in Table 3[**] , and principal component loadings are shown in Table 4.

Table 3: *Principal component standard deviations, variance explained, and cumulative variance explained for grade dataset*

Importance of Components	PC1	PC2	PC3	PC4	PC5	PC6
Standard Deviation	2.1617	0.8501	0.60810	0.36715	0.28017	0.14616
Proportion of Variance	0.7788	0.1205	0.06163	0.02247	0.01308	0.00356
Cumulative Proportion	0.7788	**0.8993**	0.96089	0.98336	0.99644	1.00000

As Table 3 shows, the first principal component (PC1) and the second principal component (PC2) preserve 89.93% of the original data's variability. Therefore, we could reduce the dimension of our data from six to two dimensions by PC1 and PC2. We applied clustering methods to PC1 and PC2 data. Three clusters are shown in Figure 4, which presents a graph drawn on a two-dimensional coordinate plane.

[**] Analysis data and R code can be found at http://math1.skku.ac.kr/home/pub/665/.

Table 4: *Principal component loadings for grade dataset*

	PC1	PC2	PC3	PC4	PC5	PC6
Final	-0.4500	0.0352	0.2949	-0.2186	0.0416	0.8123
PBL	-0.4382	0.0768	0.3328	-0.5931	0.2239	-0.5380
Mid	-0.4233	-0.1065	0.4895	0.6509	-0.3154	-0.2163
Q&A	-0.4038	-0.3687	-0.5301	-0.2321	-0.6032	-0.0468
Atte	-0.4225	-0.2648	-0.4165	0.3220	0.6883	-0.0199
Project	-0.2911	0.8806	-0.3288	0.1388	-0.1049	-0.0373

PC1 preserves the most original data. Since the loadings of PC1 were all negative, as in Table 4, the lower the PC1 value, the higher the achievement. Figure 4 also shows the grade given based on the PC1 scores. Students who got grade F were grouped into Group 3, while grades A, B, and C were mixed in Groups 1 and 2. It is consistent with the actual grades from the original data.

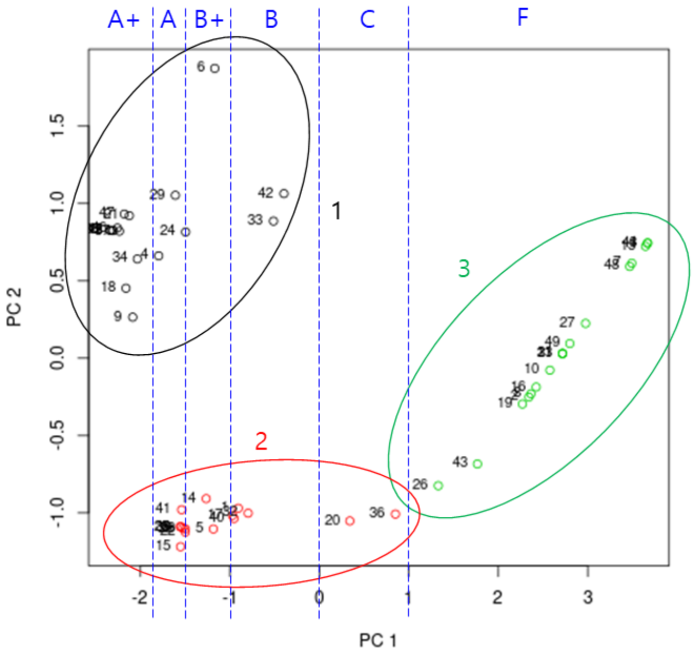

Figure 4: *Clustering result with three clusters and Tentative grade*

In our online class, all the PC1 loadings were negative, but their absolute values were generally evenly spread out within the range of approximately 0.2 to 0.4. However, the PC2 loadings were mixed with negative and positive numbers. The project was the most influential factor in the absolute value of the loading (approximately 0.88).

When students finalized the project voluntarily and presented it to others, most of them got recognition from peers for the presentation. Extra points measure their recognition. Whenever there were extra points, their PC2 values increased. Those students were classified in Group 1. The biplot expresses the relationship between the original variables and principal components in Figure 5 shows that the PC1 is a linear combination of the portfolio, final and midterm exams, attendance, and Q&A factors. At the same time, the PC2 comes from the project video presentation factor (Figure 5). The data and results confirm that the factor determining whether students belong to Groups 1 and 2 was the project video presentation.

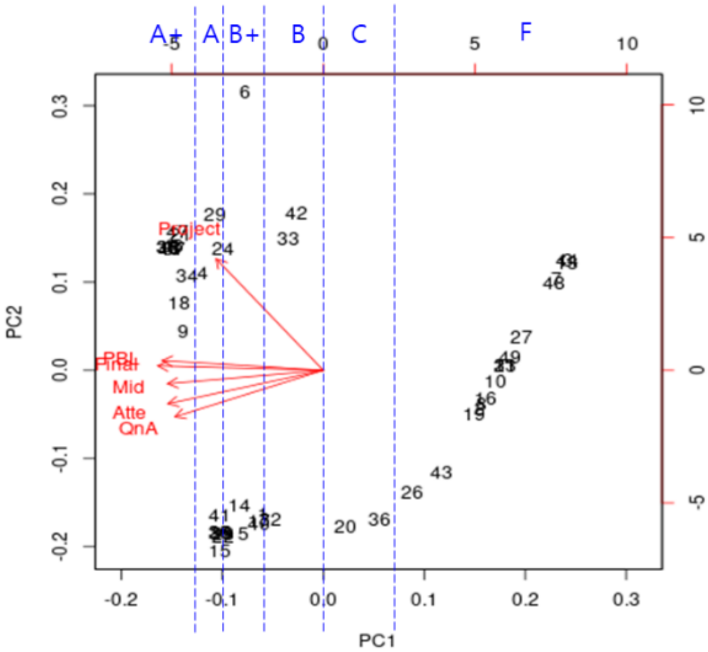

Figure 5: *Biplot expressing the relationship between the original variables and P.C.s*

The project presentation made a difference in final grades, resulting in Groups 1 and 2. We interpret this as the difference in students' achievement from this course. It might not be possible to conclude that the project video presentation is significantly more critical, and the students with good project video presentation understand content completely. However, it seems clear that the project video presentation was a critical factor in deciding peer reviewers' confidence and final grade.

4.2 Analysis of course evaluation

We analyzed the survey/course evaluation of students. Table 5 summarizes the results as follows. Most of the items scored excellent satisfaction, exceeding 4 out of 5 points. In particular, the highest average score was for the question "I was able to gain new knowledge and perspectives in this class," indicating that most students were positively aware that they were able to acquire new knowledge and perspectives on mathematics for AI.

Table 5: *Course evaluation data on class satisfaction*

	Questions	Strongly agree	Agree	Neither agree nor disagree	Dis-agree	Strongly disagree	Achiev-ement
1	I was able to gain new knowledge and perspectives in this class.	24 (71%)	0 (0%)	8 (24%)	1 (3%)	1 (3%)	4.32
2	Assignments (Q&A and Portfolio), readings, papers, quizzes, and exams were pertinent to the subject and enhanced learning.	23 (68%)	4 (12%)	3 (9%)	2 (6%)	2 (6%)	4.29
3	Attending this course expanded my knowledge and gave me better insights into the subject matter.	24 (71%)	1 (3%)	6 (18%)	1 (3%)	2 (6%)	4.29

In addition, students reported that the instructor ran the course as specified in the syllabus, according to the Faculty/Course Evaluation (Table 6). Therefore, based on this score, we could conclude that there was no difficulty following the learning path. Moreover, it seems that students invested time and effort in this course. They acknowledged that various assignments and evaluation methods used in the class were related to the subject and improved their knowledge. Communication with instructors was judged to have taken place smoothly. Most importantly, students recognized that they had expanded their knowledge and gained insight into mathematics for AI and its application to their fields.

Table 6: *Faculty/course evaluation*

	Questions	Satisfaction
1	I spent more effort and time on this class than any other courses for which I was registered this semester.	3.97
2	The instructor conducted the course as outlined in the syllabus throughout the semester.	4.47
3	I was able to gain new knowledge and perspectives in this class.	4.32
4	Assignments, readings, papers, quizzes, and exams were pertinent to the subject and enhanced learning.	4.29
5	The instructor's enthusiasm and commitment stimulated my motivation and increased my interest in this class.	4.15
6	A means to communicate with the instructor was always available.	4.59
7	Attending this course expanded my knowledge and gave me better insights into the subject matter.	4.29

The analysis shows that students were generally satisfied with our learning method and recognized that their knowledge and understanding of the subject were improved.

4.3 Examples of students' responses to the course

Under the situation that students cannot participate in face-to-face classes, online classes must provide a teaching and learning environment to reflect on students' educational process and manage their learning. Students' portfolios showed that our PBL course effectively provided such an environment.

Students' portfolios containing learning process, self-assessment, and individual learning outcomes, were presented every activity they performed. There were more than 3,000 activities in the Q&A, summary, review, questions, answers, discussion, sharing presentation, and projects. Students' reflecting learning process in the Q&A is shown in Table 7.

Table 7: *PBL (portfolio) report*

Midterm portfolio	http://matrix.skku.ac.kr/2020-Mid-PBL-1/ http://matrix.skku.ac.kr/2020-Mid-PBL-2/ http://matrix.skku.ac.kr/2021-Fall/MidtermPBL/
Final portfolio	http://matrix.skku.ac.kr/Math4AI-Summary/ http://matrix.skku.ac.kr/2020-Math4AI-Final-pbl2/ http://matrix.skku.ac.kr/2021-Final-PBL-E/ http://matrix.skku.ac.kr/2021-Math4AI-Fall-PBL/

They found they had acquired a good habit of organizing lecture content by actively participating in the Q&A activities every week. We believe that it satisfied the purpose of the study—to strengthen students' self-learning ability. Some students gave the following feedback:

> *"This PBL system pushes me to learn and improve my knowledge literally every day. So, every day I devoted at least 3-4 hours to study this subject and related material, which allowed me to significantly improve my mathematical knowledge and practical skills in programming in SageMath, Python and R."*

> *"I believe that all this was made possible through the use of the PBL learning system. I also understood the importance of the active participation of students in the educational process. I believe that thanks to the active participation of students and instructor in the Q&A, many even rather complex mathematical topics have become much clearer."*

As shown in the students' responses below, they could manage their studies well. In addition, they participated more actively in the class through the 'Learning Process Diagnosis' True/False-type quiz questions provided by the LMS of this study.

> *"I am very pleased that most of the students in this class are very actively involved in the educational process. Everyone is trying to share their knowledge in Q&A, ask questions on*

> *an incomprehensible topic, or help another student with a detailed answer. I also try to be a part of this process and to participate as much as possible in the Q&A."*

> *"As the class progressed, it became a habit to adjust and check the inquiry board every day gradually. I mainly feel the joy of communicating while taking this class."*

Students reported they adapted well to the new teaching and learning method, and in particular, they were delighted with the online mathematics practice. Their confidence in solving complex problems increased because of the given Lab. They also felt their coding ability had improved.

> *"I understood at a sufficient level most of the topics from Part 0 to Part 4. I also learned many new methods of solving mathematical problems using programming languages (SageMath, Python)."*

Students sufficiently studied and understood the mathematical knowledge for AI in the course. In addition, students realized that their knowledge had become more concrete as they clarified their understanding of various complex mathematical concepts during the discussion and presentation.

> *"Over the past three weeks, I have greatly improved my knowledge of statistics, and most importantly, I learned to apply this knowledge to solve real problems. From other colleagues, I learned a lot about Linear Algebra, Calculus, Statistics, and thanks to other students, I learned many new ways of applying mathematical knowledge in practice, in particular, in the field of artificial intelligence. Many students were able to help each other understand a variety of difficult mathematical topics. And I also think that thanks to the active participation of students, the instructor, and T.A. in the learning process, many students were able to master most of the material from all four parts at a fairly good level. By their active participation in the educational process, students have become a solid support for each other, which helps to quickly and relatively easily master this rather voluminous subject."*

The following was the final comment after the final PBL report/exam given by a student:

> *"The first eight weeks were beneficial for me. I got to revise all my past knowledge about math and had a chance to discuss different math topics. I learned a lot of new things about matrices, and as a student with the Software as a Major, I got a more thorough understanding of math concepts behind the AI. The rest of the weeks were excellent. The second half was fascinating while we were just preparing to learn AI during the first eight weeks. Each lecture had so much helpful material that is very important to understand AI and its logic. Now I can talk about*
>
> *Singular Value Decomposition (SVD)*
> *Gradient Descent Method*
> *Data and Covariance Matrix*
> *Principal Components Analysis (PCA)*
> *Rank Reduction and the role of SVD in PCA*
> *Backpropagation algorithm in Machine Learning and Artificial Neural Network*
>
> *Overall, it has been fun learning this course. Due to the instructor who focuses on having an enjoyable yet effective class, I have been able to learn without any pressure and while having fun. I have realized the importance of participation in Q&A after being actively involved in it. The participation has helped me to increase my knowledge and clear my confusion. The*

lectures are informative and well organized. The best thing about the lecture is how the instructor summarized what we would study before the lecture and what we had studied after the lecture. It helps us remember and organize what we will learn and what we have learned during the lecture. The lectures got more interesting after the mid-term. There were new yet fascinating topics that we discussed. The lectures and materials provided were very informative. From experience before mid-term, I got the hang of how I should participate in Q&A and did it accordingly. I am very grateful for that course and believe that everything I learned here will be very helpful in the future. I have enjoyed learning this course and look forward to taking another course taught by our instructor."

4.4 Examples of instructor's responses

Our method significantly improved performance from the 15-week *Introductory Mathematics for AI* online course in 2020. The opinions of instructors supporting this conclusion are as follows.

1. Students were asked to finalize questions or discussions themselves and record results in their portfolios in this class. We think that this activity increased students' participation in the course.
2. Students were asked to share their experience on improving their insufficient areas in their respective reports. In addition, the instructor gave enough opportunities for reinforcement before the final exam. As a result, underdeveloped students would not give up in the middle of the course with the help of classmates.
3. Our method of evaluation of the learning activity was sufficient and good enough to give a fair evaluation for scores of A, B, C, and D. It seemed the students who completed the project achieved the most remarkable results. In addition, interim grade notifications and opportunities for reinforcements resulted in almost no complaints about their grades.
4. By providing an online practice lab that allows simulation and calculation even on mobile phones, students could learn the necessary mathematics subjects in a short time and proceed to deal with more advanced level problems. It made students reach a higher level of learning achievement. It was possible to build confidence in not only AI mathematics but also coding.

5. Conclusion

In the era of the fourth industrial revolution, interest in AI has been increasing in the field of education [1,9]. In addition, non-contact education has become a standard. Our research interest focuses on maximizing the advantages of non-contact education and overcoming its environmental limitations [8]. While it is necessary to reflect the demand for the latest technologies, such as AI and big data in our curriculum, developing and discovering teaching and learning cases suitable for the online education method is extremely important [21].

This paper presented our 15 weeks curriculum that combines AI and mathematics for first-year college students. We aimed to make students understand how to utilize data and infer results using AI. To do that, we had to make them use mathematical language and analyze real-life data. The *Introductory Mathematics for AI* course consisted of linear algebra, multivariate calculus, statistics, and PCA/ML with computer program coding practice. Notably, the course was structured so that students with diverse backgrounds could learn basic mathematics for AI and connect it to their areas of interest. Finally, students had the chance to analyze a real-life MNIST data set.

We provided an online mathematics laboratory using Python (Python/SageMath) and R languages to train students to analyze data. Mathematics learning content and environmental change inevitably require many engineering tools and practice to solve real-world problems in the learning process

[14,28]. Our mathematics laboratory shortens the time and effort required to understand complicated mathematical concepts and perform calculations. In addition, it provides an environment to improve student's learning efficiency and application ability with real-life data analysis on the platform.

Moreover, the course was designed to consider the following four areas: change in the interaction between instructors and students in non-contact education, sharing and collaboration, reinforcement of self-learning management, and fairness of evaluation.

First, the LMS was activated in various ways to reinforce self-learning capabilities in this study. Our method supported students' personalized level-specific learning. Students could participate and broaden their understanding of mathematics needed for AI while working together virtually to solve problems through this environment.

To prevent the lack of interaction between the instructor and students and help students progress to an advanced level of knowledge, the Q&A of the LMS was actively used in the class. Newly developed True/False quiz questions helped students' learning process. In addition, students' portfolios improved their self-learning management. We adopted a process-driven evaluation method to pursue the fairness and reliability of the evaluation.

After analyzing the class activities by the PCA technique, we confirmed that various factors were evenly reflected in the evaluation. According to education experts, online evaluations need to be designed more systematically and diverse than offline evaluations [7]. Researchers have recommended gathering ideas to conduct evaluations in any form and provide the necessary platforms. Our study demonstrates that our method works well and that students achieved more. We conclude that the evaluation elements were well applied to the students of each group. Thus, the elements of the evaluation set contributed to the validity of the online evaluation.

This study's survey/course evaluation shows that students recognized their highly improved new knowledge and perspective of the subject. Students broadened their knowledge and gained insights into their majors. They demonstrate that students were aware that they had broadened their knowledge of mathematics for AI and gained insights into their interests. Moreover, students showed a positive perspective in the course evaluation, including communication with the instructor and the overall class. One of the goals of this study was to reinforce self-learning management ability in the online environment. It appears that students could manage self-learning through Q&A activities, True/False quizzes, and portfolios containing all of these activities.

In the face of a disrupted educational environment due to the pandemic in recent years, the effort has to be made to discuss and solve each education community's issues and foresee future problems [8]. We consider the case of the online *Introductory Mathematics for AI* course using our method introduced in this study as a reasonable and adequate teaching model applicable to other online courses.

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Appendix

A.1. Sample lectures and PBL report format for the course https://buk.io/@kb5299

[Sample Lectures]	
Syllabus	https://youtu.be/F9B2xooZvQY
What is AI?	https://youtu.be/F1HNFGAMhro
Least Square Solution	https://youtu.be/pYQJdE8A1FQ https://youtu.be/3DwPr0rq5wA
Singular Value Decomposition	https://youtu.be/3PYdsQKLEm4
Gradient Descent Method	https://youtu.be/IADbbl_iAwI https://youtu.be/zUpZa_WPoFA
Covariance Matrix	https://youtu.be/MSu5_ehP_Ug
PCA and Dimension Reduction	https://youtu.be/Skyi06WVRgA https://youtu.be/5j6Se97OTA8
ANN and Backpropagation	https://youtu.be/T6OSpzz00HU https://youtu.be/z1bmkRhH050
[PBL Report Format]	http://matrix.skku.ac.kr/PBL-Form/PBL.hwp http://matrix.skku.ac.kr/PBL-Form/PBL-Report-Form-English.docx
Midterm PBL Report (example)	http://matrix.skku.ac.kr/2020-Mid-PBL-1/ http://matrix.skku.ac.kr/2020-Mid-PBL-2/
Final PBL Report (example)	http://matrix.skku.ac.kr/2020-math4ai-final-pbl/ http://matrix.skku.ac.kr/2020-Math4AI-Final-pbl2/ http://matrix.skku.ac.kr/2021-Final-PBL-E/ http://matrix.skku.ac.kr/2021-Final-PBL/

A.2. Lecture notes and online labs for the course

Part	Theme	http://matrix.skku.ac.kr/intro-math4ai/
I. Basic Math for AI	Function, Graph, and Solution of Equations	http://matrix.skku.ac.kr/intro-math4ai/W1/
II. AI and Matrix	Data and Matrixes	http://matrix.skku.ac.kr/intro-math4ai/W2/
	Classification of Data	http://matrix.skku.ac.kr/intro-math4ai/W3/
	System of Linear Equations	http://matrix.skku.ac.kr/intro-math4ai/W4/
	Least Squares Problem	http://matrix.skku.ac.kr/intro-math4ai/W5/
	Matrix Decompositions	http://matrix.skku.ac.kr/intro-math4ai/W6/
III. AI and Optimal Solution	Limits of Functions	http://matrix.skku.ac.kr/intro-math4ai/W7/
	Local Maximum and Minimum	http://matrix.skku.ac.kr/intro-math4ai/W8/
	Gradient Descent Method	http://matrix.skku.ac.kr/intro-math4ai/W9/
IV. AI and Statistics	Counting Method, Probability, Random Variable, Bayes' Theorem	http://matrix.skku.ac.kr/intro-math4ai/W10/
	Expectation, Variance, Correlation Coefficient, Covariance Matrix	http://matrix.skku.ac.kr/intro-math4ai/W11/
V. PCA and ANN	Principal Component Analysis	http://matrix.skku.ac.kr/intro-math4ai/W12/
	Artificial Neural Network	http://matrix.skku.ac.kr/intro-math4ai/W13/
	Hand-written Numbers Detection	http://matrix.skku.ac.kr/intro-math4ai/W14/